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INSTABILITY ANALYSIS PROCEDURE FOR THREE-LEVEL MULTI-BEARING ROTOR-FOUNDATION SYSTEMS

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A procedure for the instability analysis of three-level multi-span rotor systems is described. This procedure is based on a distributed mass-elastic representation of the rotor system in several eight-coefficient bearings. Each bearing is supported from an elastic foundation on damped, elastic pedestals. The foundation is represented as a general distributed mass-elastic structure on discrete supports, which may have different stiffnesses and damping properties in the horizontal and vertical directions. This system model is suited to studies of instability threshold conditions for multi-rotor turbomachines on either massive or flexible foundations. The instability condition is found by obtaining the eigenvalues of the system determinant, which is obtained by the transfer matrix method from the three-level system model. The stability determinant is solved for the lowest rotational speed at which the system damping becomes zero in the complex eigenvalue, and for the whirl frequency corresponding to the natural frequency of the unstable mode. An efficient algorithm for achieving this is described. Application of this procedure to a rigid rotor in two damped-elastic bearings and flexible supports is described. A second example discusses a flexible rotor with four damped-elastic bearings. The third case compares the stability of a six-bearing 300 Mw turbine generator unit, using two different bearing types. These applications validate the computer program and various aspects of the analysis.

INTRODUCTION

Problems of rotor instability continue to occur in modern turbomachinery as stability limits for speed, power, and flexibility are pressed more closely by advanced rotating equipment. Established methods for raising the instability threshold speed, such as flexible supports, stable bearing types, etc., are sometimes unable to accommodate operational requirements imposed by such designs. At the same time, demands for simpler and less costly support structures can introduce additional vibration problems to the turbomachine-foundation structure, which may further influence the instability threshold speed of the rotor system.

The purpose of the paper is to describe a general-purpose multi-bearing rotordynamics computer code for the calculation of instability whirl threshold speeds for three-level rotor systems, of the type shown in figure 1. The rotor has distributed mass-elastic properties, and may carry massive disks at the end of each shaft section. The usual linear eight-coefficient representation of bearing dynamic

properties is employed at each support location, and each bearing may be mounted upon a massive pedestal which is flexibly supported with damping from the foundation structure. The foundation also has distributed mass-elastic properties with allowance for concentrated masses at the ends of each foundation section. The foundation is mounted on discrete, damped, flexible supports which attach it to ground. The pedestal supports and the foundation supports both attach to the foundation, but not necessarily at the same locations: see figure 1. In this manner the influence of massive pedestals, and of a flexible, tuned foundation on the instability threshold of a multi-bearing rotor may be obtained.

NOMENCLATURE

A	Cross sectional area of shaft
c	Radial bearing clearance
C_{px}, C_{py}	Pedestal damping coefficient
C_{xx}, C_{xy}, C_{yx}, C_{yy}	Bearing damping coefficient
E	Young's modulus
F_x, F_y	Bearing reaction forces
I	Cross sectional moment of inertia
I_p	Polar mass moment of Inertia
I_T	Transverse mass moment of inertia
K_{px}, K_{py}	Pedestal stiffness coefficient
K_{xx}, K_{xy}, K_{yx}, K_{yy}	Bearing stiffness coefficients
L_n	Length of shaft section between station n and n+1
M_n	Bending moment to the left of station n
M'_n	Bending moment to the right of station n
M_{px}, M_{py}	Pedestal mass components
m_n	Mass at rotor station n
t	Time
V_n	Shear force to the left of station n
V_n'	Shear force to the right of station n
W	Bearing static load
x, y	Components of rotor whirl displacement
z	Axial coordinate along rotor
Θ_x, Θ_y	Components of deflected rotor slope
ω	Angular frequency of rotor
ω_n	Critical frequency of rotor
f_{rth}	Frequency ratio $= \omega_w / \omega_{th}$
Δ	Determinant of matrix
Δ_c	Real part of determinant

Δ_s	Imaginary part of determinant
ω_{th}, N_{th}	Threshold frequency, threshold speed
ω_w	Whirl frequency ($=\omega_{th} \times f_{rth}$)

PREVIOUS WORK

Studies of the instability properties of multi-bearing rotor systems are comparatively rare in the open literature. The earliest rotordynamic analysis of such systems appears to date back to Borowicz [1] in 1915, while numerical procedures for multi-rotor systems were initiated by Prohl [2] in 1945 for critical speeds, and by Koenig [3] in 1961 for multi-rotor system unbalance response studies with damped flexible bearings. More advanced multi-bearing rotor system procedures and codes were developed by Lund and others, for unbalance response and instability analysis of discrete rotor models [4], and for analysis of distributed mass-elastic rotors [5] and [6]. Critical speeds of multi-rotor systems with flexible bearing were also studied by Crook and Grantham [7]. Other multi-bearing rotordynamic codes were developed by Shapiro and Reddi [8], Zorzi and Nelson [9] using the finite element method, and by Gunter [10]. Most of these studies have included the influence of massive flexible pedestals, but there do not appear to have been any published studies in which the influence of pedestals and a general mass-elastic foundation on flexible supports is included. Most studies have concentrated on the influence of multiple spans on critical speeds and unbalance response, or on the influence of massive flexible supports on such properties. The only previous multi-bearing flexible support study which deals with instability threshold speed appears to be that by Lund [6]. The code upon which Lund's study is based includes pedestal effects, but not distributed foundation effects. Lund's study is noteworthy for the experimental confirmation of the theoretical threshold speed predictions which it includes.

SYSTEM MODEL

The system model used in this analysis is shown in figure 1. This model allows details of the rotor, bearings, pedestals, distributed foundation, and foundation supports to be included in the system response results. The number of rotor stations and the number of bearing and foundation supports, is limited only by the computer space available. Rotor stations and foundation stations are independent, and may be included as required within the system model.

The rotor is massive and elastic and its internal damping is small enough to be neglected. As the rotor is symmetric about its axis of rotation, it is modeled using circular cylindrical sections having mass, elasticity, rotatory and polar inertia properties distributed along their length. Both transverse bending and shear effects contribute to the rotor stiffness. The rotor is referred to as the first level of the system. For analysis a rotor is divided into uniform shaft sections, and large disks with concentrated mass and inertia properties are included at rotor stations at the ends of the shaft sections, i.e., at the stations designated 1, 2, 3, etc., in figure 1. For instability studies the rotor unbalance is omitted for this model.

The bearings which support the rotor at specified stations are represented by the well-known eight stiffness and damping coefficient procedure, which includes

both direct and cross-coupled relative displacement and relative velocity effects, between the journal and the pedestal motions. The bearing properties are therefore linear for small displacements, in keeping with the rest of the structural components. The pedestals which support the bearings are represented by mass properties in the x- and y- directions (no inertia properties), and they are supported by elastic, damped members in these directions. This model allows two-dimensional transverse pedestal dynamics (without support coupling) to be included. The pedestals are the second level of the system model.

The foundation is a continuous structural member which has different mass-elastic properties in the x- and y- directions. It is modeled in a similar manner to the rotor, using prismatic shear-beam members with distributed properties between stations. Discrete mass effects such as casing mass, generator stator, gearbox, etc., are incorporated at the end stations. The foundation supports are represented by concentrated elastic-damping elements at the ends of the beam sections, as required, in a similar manner to the rotor bearing-pedestal supports. The foundation supports possess different stiffness and damping properties in the x- and y- directions. The foundation is the third level of the system model.

FORCES AND MOMENTS ON ROTOR STATIONS

The convention and notation used in this analysis are shown in figure 2, for the x-z plane. Similar expressions apply for the y-z plane. The forces and moments which act on the n-th rotor station are shown in figure 3. They are considered in the following manner.

a. Inertia forces. The inertia forces which act on the concentrated mass of the rotor station at angular frequency ω are given by:

$$\begin{aligned} \Delta V_{xM} &= - m \frac{\partial^2 x}{\partial t^2} = m \omega^2 x \\ \Delta V_{yM} &= - m \frac{\partial^2 y}{\partial t^2} = m \omega^2 y \end{aligned} \tag{1}$$

b. Inertia moments. The inertia moments which act on the concentrated translatory and polar inertias at the rotor station at angular frequency ω are given by:

$$\begin{aligned} \Delta M_{xM} &= (I_p - I_T) \omega^2 \theta_x \\ \Delta M_{yM} &= (I_p - I_T) \omega^2 \theta_y \end{aligned} \tag{2}$$

c. Bearing reactions. Each bearing is represented by eight stiffness and damping coefficients in the customary manner. The bearing forces are given by:

$$\begin{aligned} \Delta V_{xB} &= - K_{xx} (x - x_p) - C_{xx} (\dot{x} - \dot{x}_p) - K_{xy} (y - y_p) - C_{xy} (\dot{y} - \dot{y}_p) \\ \Delta V_{yB} &= - K_{yx} (x - x_p) - C_{yx} (\dot{x} - \dot{x}_p) - K_{yy} (y - y_p) - C_{yy} (\dot{y} - \dot{y}_p) \end{aligned} \tag{3}$$

Here x_p and y_p are the pedestal displacements, and $\dot{x}_p$ and $\dot{y}_p$ are the pedestal velocities. The cross-coupling terms in these expressions couple the bearing and pedestal displacements and velocity. The bearing coefficients are functions of Sommerfeld number, bearing type, and Reynolds number if turbulent flow is involved. The influence of bearing moments on the rotor is usually small and is neglected in this analysis.

FORCES AND MOMENTS ON SHAFT SECTIONS

The analysis procedure for the shaft follows the distributed mass-elastic method described by Lund and Orcutt [5], with the difference that the harmonic unbalance components are zero in the fourth-order equations of motion. This procedure is not repeated here, but is similar to that given in the next section for the foundation analysis.

FORCES AND MOMENTS ON FOUNDATION SECTIONS

A section of the foundation between stations r and r+1 is shown in figure 4. The foundation analysis is performed by developing a transfer matrix for each section, in a similar manner to the rotor analysis. The structure is sub-divided into uniform shear-beam sections. All speed-dependent terms in the foundation equations are zero, and the foundation is allowed to have different stiffness properties in the horizontal and vertical directions. The equations of motion for a uniform foundation beam section between end stations in the x- and y-directions are

$$EI_x \frac{\partial^4 x_f}{\partial z^4} - \rho I_x \left\{ \frac{E}{\alpha_x G} + \frac{I_{Tx}}{\rho I_x} \right\} \frac{\partial^4 x_f}{\partial z^2 \partial t^2} + \rho A \frac{\partial^2 x}{\partial t^2} + \frac{\rho I_{Tx}}{\alpha_x G} \frac{\partial^4 x_f}{\partial t^4} = 0 \qquad (4a)$$

$$EI_y \frac{\partial^4 y_f}{\partial z^4} - \rho I_y \left\{ \frac{E}{\alpha_y G} + \frac{I_{Tx}}{\rho I_y} \right\} \frac{\partial^4 x_f}{\partial z^2 \partial t^2} + \rho A \frac{\partial^2 x}{\partial t^2} + \frac{\rho I_{Tx}}{\alpha_y G} \frac{\partial^4 x_f}{\partial t^4} = 0 \qquad (4b)$$

To solve these expressions set

$$x_f = \bar{x}_f e^{i\omega t} \qquad (5)$$

$$y_f = \bar{y}_f e^{i\omega t}$$

Substituting gives the expressions

$$EI_x \frac{d^4\bar{x}_f}{dz^4} + \rho I_x \omega^2 \{ \frac{E}{\alpha_x G} + \frac{I_{Tx}}{\rho I_x} \} \frac{d^2\bar{x}_f}{dz^2} - \rho A \omega^2 \{1 - \frac{\omega^2 I_{Tx}}{\alpha_x GA} \} \bar{x}_f = 0 \qquad (6a)$$

$$EI_y \frac{d^4\bar{y}_f}{dz^4} + \rho I_y \omega^2 \{ \frac{E}{\alpha_y G} + \frac{I_{Ty}}{\rho I_y} \} \frac{d^2\bar{y}_f}{dz^2} - \rho A \omega^2 \{1 - \frac{\omega^2 I_{Ty}}{\alpha_y GA} \} \bar{y}_f = 0 \qquad (6b)$$

The form of these equations is identical, and they are uncoupled because of the coordinate symmetry. Where $I_x \neq I_y$ the x- and y- equations must be solved separately. To demonstrate the solution procedure for the x- direction, write

$$2\sigma^2\lambda^2 = \omega^2 \{ \frac{\rho}{\alpha G} + \frac{I_T}{EI} \}; \qquad \lambda^4 = \frac{\rho A \omega^2}{EI} \{ 1 - \frac{\omega^2 I_T}{\sigma GA} \} \qquad (7)$$

Substitution in equation 6a gives

$$\frac{d^4\bar{x}_f}{dz^4} + 2\sigma^2\lambda^2 \frac{d^2\bar{x}_f}{dz^2} - \lambda^4\bar{x}_f = 0 \qquad (8)$$

The solution to this expression is

$$\bar{x}_f = D_1 \cosh \lambda_1 z + D_2 \sinh \lambda_1 z + D_3 \cos \lambda_2 z + D_4 \sin \lambda_2 z \qquad (9)$$

where D_1, D_2, D_3, and D_4 are integration constants to be determined from the boundary conditions. The eigenvalues of these expressions are given by

$$\lambda_1 L_f = \lambda L_f \{ \sqrt{1 + (\sigma\lambda)^4} - (\sigma\lambda)^2 \}^{\frac{1}{2}}$$
$$\lambda_2 L_f = \lambda L_f \{ \sqrt{1 + (\sigma\lambda)^4} + (\sigma\lambda)^2 \}^{\frac{1}{2}} \qquad (10)$$

The boundary conditions for the foundation section are

$$z = 0;\ x_f = x_{fn},\ \theta = \theta_{fn},\ M_f = M_{fn},\ V_f = V_{fn}$$

At the other end of the section $z = L_{fn}$; $x_f = x_{f,n+1}$, $\theta_f = \theta_{f,n+1}$, $M_f = M_{f,n+1}$, $V_f = V_{f,n+1}$.[1] Upon substitution the required transfer matrix expressions for the foundation become:

[1]The simplest procedure is to eliminate the distributed rotary inertia effects I_{Tx}, I_{Ty} and to account for these terms by using lumped inertia where necessary at the end of the foundation section. Thus we will neglect I_{Tx} and I_{Ty}.

$$x_{f,n+1} = b_{1n}\, x_{f,n} + L_{f,n}\, b_{3n}\, \Theta_{x,n} + h_{2n}\, b_{4n}\, M'_{f,n} + h_{3n}\, b_{7n}\, V'_{f,n}$$

$$\Theta_{f,n+1} = \frac{1}{3}\phi_n\, h_{2n}\, b_{5n}\, L_{f,n} + b_{2n}\, \Theta_{x,n} + h_{1n}\, b_{6n}\, M'_{f,n} + h_{2n}\, b_{4n}\, V'_{f,n}$$

$$M_{f,n+1} = \frac{1}{2}\phi_n\, L_{f,n}\, b_{4n}\, x_n + \frac{1}{6}\phi_n\, L^2_{f,n}\, b_{5n}\, \Theta_{f,n} + b_{2n}\, M'_{f,n} + L_{f,n}\, b_{3n}\, V'_{f,n} \tag{11}$$

$$V_{f,n+1} = \phi_n\, b_{3n}\, L_{f,n} + \frac{1}{2}\phi_n\, L_{f,n}\, b_{4n}\, \Theta_{f,n} + \frac{1}{3}\phi_n\, h_{2n}\, b_{5n}\, M'_{f,n} + b_{1n}\, V'_{f,n}$$

where $b_{1n} = (\lambda_1^2 \cosh \lambda_1 L_{f,n} + \lambda_2^2 \cos \lambda_2 L_{f,n}) \,/\, (\lambda_1^2 + \lambda_2^2)$

$$b_{2n} = (\lambda_2^2 \cosh \lambda_1 L_{f,n} + \lambda_1^2 \cos \lambda_2 L_{f,n}) \,/\, (\lambda_1^2 + \lambda_2^2)$$

$$b_{3n} = (\lambda_1 \sinh \lambda_1 L_{f,n} + \lambda_2 \sin \lambda_2 L_{f,n}) \,/\, (\lambda_1^2 + \lambda_2^2)\, L_{f,n}$$

$$b_{4n} = 2\,(\cosh \lambda_1 L_{f,n} - \cos \lambda_2 L_{f,n}) \,/\, (\lambda_1^2 + \lambda_2^2)\, L^2_{f,n} \tag{12}$$

$$b_{5n} = 6\,(\lambda_2 \sinh \lambda_1 L_{f,n} - \lambda_1 \sin \lambda_2 L_{fn}) / (\lambda_1^2 + \lambda_2^2)\, \lambda^2 L^3_{f,n}$$

$$b_{6n} = (\lambda_2^3 \sinh \lambda_1 L_{f,n} + \lambda_1^3 \sin \lambda_2 L_{f,n}) / (\lambda_1^2 + \lambda_2^2)\, \lambda^2 L_{f,n}$$

$$b_{7n} = 6\,(\lambda_1^3 \sinh \lambda_1 L_{f,n} - \lambda_2^3 \sin \lambda_2 L_{f,n}) \,/\, (\lambda_1^2 + \lambda_2^2)\, \lambda^4 L^3_{f,n}$$

$h_{1n} = L_{f,n}/EI$; $h_{2n} = L^2_{f,n}/2EI$; $h_{3n} = L^3_{f,n}/6EI$

$\varphi_n = \omega^2 \rho A L_{f,n}$

The solution for the foundation in the y-direction is similar to the above.

THREE-LEVEL SYSTEM

Details of the rotor-pedestal-foundation models are shown in figure 5. Using the notation shown, the equations governing the harmonic motion of this system in the x-z plane are:

$$\begin{Bmatrix} F_{xcn} \\ F_{xsn} \end{Bmatrix} = \begin{bmatrix} K_{xx} & \omega C_{xx} & K_{xy} & \omega C_{xy} \\ -\omega C_{xx} & K_{xx} & -\omega C_{xy} & K_{xy} \end{bmatrix} \begin{Bmatrix} x_{cn} - x_{cnp} \\ x_{sn} - x_{snp} \\ y_{cn} - y_{cnp} \\ y_{sn} - y_{snp} \end{Bmatrix} \tag{13}$$

$$\begin{Bmatrix} F_{xcnf} \\ F_{xsnf} \end{Bmatrix} = \begin{bmatrix} K_{xp} & \omega C_{xp} \\ -\omega C_{yp} & K_{yp} \end{bmatrix} \begin{Bmatrix} x_{cnp} - x_{cnf} \\ x_{snp} - x_{snf} \end{Bmatrix} \tag{14}$$

$$\begin{Bmatrix} F_{xcnf} - F_{xcn} \\ F_{xsnf} - F_{xsn} \end{Bmatrix} = \begin{bmatrix} M_{px} & 0 \\ -0 & M_{px} \end{bmatrix} \begin{Bmatrix} x_{cnp} \\ x_{snp} \end{Bmatrix} \tag{15}$$

where ω is the rotor exciting frequency. Similar expressions apply in the y-z direction. These expressions are used to obtain expressions for the rotor forces F_{xcn}, etc., the foundation forces F_{xcnf}, etc., and the pedestal displacements x_{cnp}, etc., in terms of the displacements of the rotor and the foundation. The transfer matrix procedure described in the previous section is then used to obtain the system transfer matrix. Assuming that the shear force and bending moment are zero at both ends of the structure, the boundary conditions may be written:

Rotor

$$\begin{aligned} V_{xc1} &= V_{xs1} = V_{yc1} = V_{ys1} = 0 \\ M_{xc1} &= M_{xs1} = M_{yc1} = M_{ys1} = 0 \\ V'_{xcr} &= V'_{xsr} = V'_{ycr} = V'_{ysr} = 0 \\ M'_{xcr} &= M'_{xsr} = M'_{ycr} = M'_{ysr} = 0 \end{aligned} \tag{16}$$

Foundation

$$\begin{aligned} V_{xc1f} &= V_{xs1f} = V_{yc1f} = V_{ys1f} = 0 \\ M_{xc1f} &= M_{xs1f} = M_{yc1f} = M_{ys1f} = 0 \\ V'_{xcrf} &= V'_{xsrf} = V'_{ycrf} = V'_{ysrf} = 0 \\ M'_{xcrf} &= M'_{xsrf} = M'_{ycrf} = M'_{ysrf} = 0 \end{aligned} \tag{17}$$

Starting from rotor station 1, the above equations are used to solve for the rotor equilibrium as described in reference [11]. Each unknown is applied separately, by setting $x_{c1} = 1$, with the other displacements zero; then apply $x_{s1} = 1$, others zero etc., through $y_{c1f} = 1$, others = zero; $y_{s1f} = 1$, others zero. This requires a total of sixteen calculations (eight rotor x,y; eight

foundations, x,y). A 16 by 16 matrix is formed from the resulting expressions. The determinant of the matrix is the system stability determinant. The lowest speed for which this matrix becomes zero is the instability threshold speed.

In general, there is no particular relationship between the number of stations of the rotor and the number of stations of the foundation, or from which locations the rotor is supported from pedestals and foundation in relation to the foundation support locations themselves. This independent rotor/foundation support situation is incorporated within the computer code by allowing a three-level support at all rotor and foundation support locations.

INSTABILITY THRESHOLD ALGORITHM

The stability determinant contains two unknowns, the rotational speed and the whirl frequency ratio fr_1. In the general case, the eight dynamic fluid-film coefficients are functions of both ω and fr_1, so that a closed form solution is not tractable. The solution is conveniently obtained by iteration as follows.

Step 1. Obtain points for Δc vs. fr_1 and Δs vs. fr_1.

For a fixed value of ω, the real part Δc and the imaginary part Δs of the stability determinant are functions of fr_1 and so may be calculated. Several points for Δc vs. fr_1 are shown in figure 7(a), and data for Δs vs. fr_1 are shown in figure 7(b).

Step 2. Find zero points of Δc and Δs by quadratic interpolation.

As shown in figure 7(a), the program determines two frequency ratios fr_1 and fr_2 where the sign of Δc changes. Then a frequency ratio fr_3 which equals to $(fr_1 + fr_2)/2$ is applied for an additional calculation and obtain Δc of fr_3. The three points with fr_1, fr_2 and fr_3 form a quadratic curve which intersects the abscissa at f_{rcw} where Δc is zero. Similarly, in figure 7(b), the zero point of Δs is obtained at frequency ratio f_{rsw}.

Step 3. Obtain points for $fc(\Delta c = 0)$ vs. ω and $fs(\Delta s = 0)$ vs. ω.

Repeat Step 1 and Step 2 with different ω until the specified speed range has been calculated. Several points for $fc(\Delta c = 0)$ vs. ω may be obtained, and similarly for $fs(\Delta s = 0)$ vs. ω, as shown in figure 7(c).

Step 4. Determine threshold speed ωth and corresponding frequency ratio f_{rth}.

Quadratic interpolation is again used. The program determines two speeds ω_1 and ω_2 where the sign of $(fc - fs)$ changes. Then a speed ω_3 which equals to $(\omega_1 + \omega_2)/2$ is applied for an additional calculation and obtain fc and fs of ω_3. The three points of fc vs. ω form a quadratic curve, and similarly for fs vs. ω. The computer program then determines the intersection point of these two curves, i.e., ω th and frth. ω th is the threshold speed where both Δc and Δs are equal to zero, and frth is the corresponding frequency ratio of ω_{th}.

Theoretically, the real parts and imaginary parts of the determinants become higher order polynomials in ω and fr, and may have many zero points for each

value of ω. But in practical use, only the zero value encountered at the lowest speed is of interest. The computer program organizes the procedure and the lowest threshold speed is obtained.

Other features of this procedure may also be described briefly as follows. There are two ways to obtain the real and imaginary parts of the determinant of a matrix. One of them is to use a complex number for each determinant calculation. But for many digital computers, this will reduce accuracy because both real and imaginary parts of the complex number have only about a half of the significant digits that the real constant has in the double precision. Another way is to use a real constant for the determinant calculation to obtain real and imaginary parts, and both parts have the same significant digits in the double precision. The latter way was used in the computer program developed here. Greater accuracy is thereby obtained.

Because a higher exponent (either positive or negative) may occur on the real and imaginary parts of the stability determinant, measures have been taken to prevent from computer overflow or underflow in the determinant calculations. Furthermore, the eight bearing coefficients are often sensitive to the results of the threshold speed, so that they should be checked carefully beforehand. A parametric study of variation of bearing coefficients on the result of threshold speed has also been performed which confirms the above comments.

EXAMPLE 1. RIGID ROTOR ON TWO BEARINGS

The cylindrical rotor shown in figure 8 has been studied by Lund [4], and is supported in damped flexible bearings and pedestals. In this instance this system has been mounted upon a continuous foundation mounted on damped flexible columns. Details are given in table 1. The threshold speed of the rotor was calculated as (a) one-level system (rotor in damped flexible bearings, rigid pedestals, and rigid foundation), (b) a two-level system (rotor in the same damped flexible bearings, very stiff pedestals, and rigid foundation), (c) as a three-level system (rotor, bearings, very stiff pedestals, and very stiff foundation and foundation supports), (d) as a three-level system (rotor, bearings, very stiff pedestals, and soft foundation and foundation supports) and (e) as a two-level system (rotor, bearings and soft pedestals). Computed results for these five cases are shown in table 2. The one-level result was checked against the one-level result given by Lund. The two-level result (with very stiff pedestals) was also checked against the same result. The analysis and results were verified through two levels against known data. The effect of very stiff pedestals in case (b) did not differ from the case (a) result. The influence of a very stiff foundation in case (c) was also found to be small. The influence of a soft foundation in case (d) was found to be large, and the effect of soft pedestals in case (e) was also found to be large, showing that the effect of pedestals and/or foundation can be very important. These results are in agreement for a rigid rotor system with a massive foundation.

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TABLE 1.- DETAILS OF THREE-LEVEL, TWO-BEARING ROTOR SYSTEM EXAMPLE

TEST: INSTABILITY, 9 STATIONS, 2 BRG. (3 LEVEL)

INSTABILITY ANALYSIS	LEVEL	FRQ-RATIO	IUNIT	IOMEGA	ICOEF	NLIST	ALIST
2	3	4	0	1	0	0	0. 00E 00

DATA FOR ROTOR:	STATIONS	BEARINGS
	9	2

DATA FOR FOUNDATION:	STATIONS	FOUNDATION SUPPORTS
	9	2

DATA OF EACH ROTOR STATION

STA	MASS	POL MOM	TRS MOM	LGTH	OD STF	ID S	OD MASS	ID M	YME7	DENS	SHME7
1	13. 6	0. 23E 03	0. 11E 03	3. 70	3. 45	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
2	31. 1	0. 51E 03	0. 25E 03	7. 75	3. 83	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
3	11. 5	0. 00E 00	0. 00E 00	8. 30	3. 61	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
4	2. 1	0. 00E 00	0. 00E 00	6. 15	3. 45	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
5	27. 6	0. 51E 03	0. 25E 03	10. 40	3. 83	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
6	43. 1	0. 12E 04	0. 58E 03	8. 65	3. 65	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
7	27. 8	0. 00E 00	0. 00E 00	5. 75	3. 45	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
8	5. 8	0. 00E 00	0. 00E 00	4. 25	3. 45	0. 00	2. 50	0. 00	3. 000	0. 283	0. 862
9	18. 2	0. 33E 03	0. 17E 03	0. 00	0. 00	0. 00	0. 00	0. 00	3. 000	0. 283	0. 862

DATA OF EACH FOUNDATION STATION

STA.	MASS	RSIXY(1)	RSIXY(2)	LGTH	AXY	YME7	DENS	SHMXE7	SHMYE7
1	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
2	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
3	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
4	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
5	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
6	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
7	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
8	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00
9	43. 1	0. 12E 11	0. 58E 10	2. 88	0. 37E 07	0. 30E 01	0. 28E 00	0. 86E 00	0. 86E 00

AT EACH OF FOLLOWING ROTOR STATION THERE IS A BEARING
2 8
AT EACH OF FOLLOWING FOUNDATION STATIONS THERE IS A FOUNDATION SUPPORT
2 8

BEARING DATA

BEARING AT STATION 2

KXX	BXX	KXY	BXY	KYX	BYX	KYY	BYY
0. 13E 06	0. 13E 07	0. 62E 06	0. 12E 06	-. 88E 05	0. 12E 06	0. 11E 06	0. 18E 06

BEARING AT STATION 8

KXX	BXX	KXY	BXY	KYX	BYX	KYY	BYY
0. 48E 06	0. 77E 06	0. 33E 06	0. 16E 06	0. 11E 05	0. 16E 06	0. 12E 06	0. 92E 05

PEDESTAL DATA

STATION	MASS-X	STIFF-X	DAMP-X	MASS-Y	STIFF-Y	DAMP-Y
2	0. 100E-17	0. 100E 11	0. 100E 03	0. 100E-17	0. 100E 11	0. 100E 03
8	0. 100E-17	0. 100E 11	0. 100E 03	0. 100E-17	0. 100E 11	0. 100E 03

FOUNDATION SUPPORT DATA

STATION	STIFF-X	DAMP-X	STIFF-Y	DAMP-Y
2	0. 100E 18	0. 100E 01	0. 100E 18	0. 100E 01
8	0. 100E 18	0. 100E 01	0. 100E 18	0. 100E 01

STATIONS WITH INTERCONNECTION BETWEEN ROTOR AND FOUNDATION, FOR ROTOR AS FOLLOW
2 8
STATIONS WITH INTERCONNECTION BETWEEN ROTOR AND FOUNDATION, FOR FOUNDATION AS FOLLOW
2 8

TABLE 1.- CONCLUDED.

```
FIRST SPEED     LAST SPEED     SPEED INCR.
0.831000E 04    0.851000E 04   0.500000E 02

FREQUENCY RATIOS
0.500000E 00    0.495000E 00   0.490000E 00   0.480000E 00
```

TABLE 2.- RESULTS OF EXAMPLE 1

Case	System	Threshold Speed, rpm		Difference
		Program	Lund [4]	
(a)	one-level	8317	8350	-0.4%
(b)	two-level, stiff pedestals	8317*		-0.4%
(c)	three-level, very stiff foun.	8204**		-1.7%
(d)	three-level, soft foundation	7217***		-13.6%
(e)	two-level, soft pedestals	7916****		-5.2%

* The two-level system with very stiff pedestals has negligible difference in threshold speed from the one-level system.

** The three-level system with very stiff foundation has a little less threshold speed than that of a one-level system.

*** The three-level system with soft foundation has a much less threshold speed than that of one-level or two-level system. This shows where the foundation effect is important.

**** The two-level system with soft pedestals has a somewhat lower threshold speed less than that of the one-level system. This also shows pedestals effect cannot be ignored.

EXAMPLE 2. FLEXIBLE ROTOR IN FOUR DAMPED FLEXIBLE BEARINGS ON A FLEXIBLE FOUNDATION

A four bearing rotor system given by Lund [4] as an unbalance response example is shown in figure 9. Details of the system are given in table 3. The rotor was modeled using a 27-station representation and calculated using the eight coefficient bearing data. A suitable foundation model was then developed and the threshold speed of the rotor was calculated (a) as a one-level system, (b) as a two-level system (with very stiff pedestals), (c) as a three-level system (with very stiff foundation), (d) as a three-level system (with soft foundation), and (e) as a two-level system (with soft pedestals). Computed results for these five cases are given in table 4. The foundation effect was found to be relatively small in case (c) and larger in case (d). Similar results were found for the pedestals effect in case (b) and (e). These results further show that in this case the effects of foundation exact a significant influence on the whirl threshold speed.

TABLE 3.- DETAILS OF THREE-LEVEL, FOUR-BEARING ROTOR SYSTEM EXAMPLE

TEST: INSTABILITY, 27 STATIONS, 4 BRG. (3 LEVEL)

INSTABILITY ANALYSIS	LEVEL	FRQ-RATIO	IUNIT	IOMEGA	ICOEF	NLIST	ALIST
2	3	7	0	1	0	0	0.00E+00

	STATIONS	BEARINGS
DATA FOR ROTOR:	27	4

	STATIONS	FOUNDATION SUPPORTS
DATA FOR FOUNDATION:	23	4

DATA OF EACH ROTOR STATION

STA.	MASS	POL. MOM	TRS. MOM	LGTH	OD. STF	ID. S	OD. MASS	ID. M	YME7	DENS	SHME7
1	76.0	0.00E+00	0.00E+00	3.75	5.72	0.00	0.00	0.00	3.130	0.283	0.900
2	21.4	0.00E+00	0.00E+00	4.09	5.62	0.00	0.00	0.00	3.130	0.283	0.900
3	30.0	0.00E+00	0.00E+00	5.31	5.62	0.00	0.00	0.00	3.130	0.283	0.900
4	125.0	0.00E+00	0.00E+00	6.62	7.36	0.00	0.00	0.00	3.130	0.283	0.900
5	124.0	0.00E+00	0.00E+00	2.25	8.86	0.00	0.00	0.00	3.130	0.283	0.900
6	382.0	0.00E+00	0.00E+00	5.91	11.22	0.00	0.00	0.00	3.130	0.283	0.900
7	876.0	0.00E+00	0.00E+00	9.78	15.14	0.00	0.00	0.00	3.130	0.283	0.900
8	777.0	0.00E+00	0.00E+00	7.19	11.43	0.00	0.00	0.00	3.130	0.283	0.900
9	570.0	0.00E+00	0.00E+00	8.57	9.47	0.00	0.00	0.00	3.130	0.283	0.900
10	453.0	0.00E+00	0.00E+00	7.19	9.41	0.00	0.00	0.00	3.130	0.283	0.900
11	235.0	0.00E+00	0.00E+00	5.00	8.99	0.00	0.00	0.00	3.130	0.283	0.900
12	300.0	0.00E+00	0.00E+00	6.62	10.57	0.00	0.00	0.00	3.130	0.283	0.900
13	313.0	0.00E+00	0.00E+00	4.87	7.92	0.00	0.00	0.00	3.130	0.283	0.900
14	82.0	0.00E+00	0.00E+00	2.13	6.10	0.00	0.00	0.00	3.130	0.283	0.900
15	32.0	0.00E+00	0.00E+00	3.19	5.40	0.00	0.00	0.00	3.130	0.283	0.900
16	36.7	0.00E+00	0.00E+00	7.42	4.49	0.00	0.00	0.00	3.130	0.283	0.900
17	28.2	0.00E+00	0.00E+00	8.65	3.25	0.00	0.00	0.00	3.130	0.283	0.900
18	90.4	0.00E+00	0.00E+00	12.38	1.59	0.00	0.00	0.00	3.130	0.283	0.900
19	4.3	0.00E+00	0.00E+00	1.24	1.59	0.00	0.00	0.00	3.130	0.283	0.900
20	4.3	0.00E+00	0.00E+00	1.24	1.59	0.00	0.00	0.00	3.130	0.283	0.900
21	78.5	0.00E+00	0.00E+00	4.75	4.49	0.00	0.00	0.00	3.130	0.283	0.900
22	34.2	0.00E+00	0.00E+00	5.32	6.87	0.00	0.00	0.00	3.130	0.283	0.900
23	98.2	0.00E+00	0.00E+00	6.19	11.48	0.00	0.00	0.00	3.130	0.283	0.900
24	243.1	0.00E+00	0.00E+00	6.51	13.44	0.00	0.00	0.00	3.130	0.283	0.900
25	330.0	0.00E+00	0.00E+00	6.21	11.42	0.00	0.00	0.00	3.130	0.283	0.900
26	284.3	0.00E+00	0.00E+00	4.33	7.50	0.00	0.00	0.00	3.130	0.283	0.900
27	63.7	0.00E+00	0.00E+00	0.00	0.00	0.00	0.00	0.00	3.130	0.283	0.900

DATA OF EACH FOUNDATION STATION

STA.	MASS	RSIXY(1)	RSIXY(2)	LGTH	AXY	YME7	DENS	SHMXE7	SHMYE7
1	0.0	0.24E+11	0.24E+11	0.45	0.55E+06	0.59E+00	0.26E+01	0.21E+01	0.21E+01
2	0.0	0.48E+11	0.48E+11	0.96	0.78E+06	0.59E+00	0.26E+01	0.21E+01	0.21E+01
3	63000.0	0.48E+09	0.48E+09	0.60	0.78E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
4	0.0	0.48E+09	0.48E+09	0.60	0.78E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
5	24000.0	0.20E+10	0.20E+10	2.04	0.16E+06	0.59E+00	0.26E+01	0.21E+01	0.21E+01
6	0.0	0.20E+10	0.20E+10	2.04	0.16E+06	0.59E+00	0.26E+01	0.21E+01	0.21E+01
7	62000.0	0.47E+09	0.47E+09	0.65	0.76E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
8	0.0	0.47E+09	0.47E+09	0.65	0.76E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
9	28000.0	0.11E+10	0.11E+10	1.95	0.12E+06	0.59E+00	0.26E+01	0.21E+01	0.21E+01
10	0.0	0.40E+09	0.40E+09	1.96	0.71E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
11	40000.0	0.47E+09	0.47E+09	0.52	0.76E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
12	0.0	0.47E+09	0.47E+09	0.52	0.76E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
13	148000.0	0.38E+10	0.38E+10	2.85	0.22E+06	0.59E+00	0.26E+01	0.21E+01	0.21E+01
14	0.0	0.12E+08	0.12E+08	2.85	0.13E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01

TEST: INSTABILITY, 27 STATIONS, 4 BRG. (3 LEVEL)

15	35000.0	0.19E+09	0.19E+09	1.63	0.48E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
16	0.0	0.54E+09	0.54E+09	0.60	0.83E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
17	0.0	0.54E+09	0.54E+09	0.60	0.83E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
18	184000.0	0.19E+09	0.19E+09	2.55	0.48E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
19	38000.0	0.31E+08	0.31E+08	4.00	0.20E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
20	3000.0	0.31E+08	0.31E+08	1.59	0.20E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
21	2000.0	0.99E+07	0.99E+07	1.45	0.11E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
22	0.0	0.26E+09	0.26E+09	0.68	0.57E+05	0.59E+00	0.26E+01	0.21E+01	0.21E+01
23	0.0	0.00E+00	0.00E+00	0.00	0.00E+00	0.59E+00	0.26E+01	0.21E+01	0.21E+01

TABLE 3.- CONCLUDED.

AT EACH OF FOLLOWING ROTOR STATION THERE IS A BEARING
3 16 22 27
AT EACH OF FOLLOWING FOUNDATION STATIONS THERE IS A FOUNDATION SUPPORT
5 9 13 17

BEARING DATA

BEARING AT STATION 3

KXX	BXX	KXY	BXY	KYX	BYX	KYY	BYY
0.15E+07	0.34E+07	-.80E+06	0.13E+07	0.97E+06	0.13E+07	0.15E+06	0.10E+07

BEARING AT STATION 16

KXX	BXX	KXY	BXY	KYX	BYX	KYY	BYY
0.15E+07	0.34E+07	-.80E+06	0.13E+07	0.97E+06	0.13E+07	0.15E+06	0.10E+07

BEARING AT STATION 22

KXX	BXX	KXY	BXY	KYX	BYX	KYY	BYY
0.15E+07	0.33E+07	-.10E+07	0.18E+07	0.12E+07	0.18E+07	-.12E+06	0.18E+07

BEARING AT STATION 27

KXX	BXX	KXY	BXY	KYX	BYX	KYY	BYY
0.15E+07	0.33E+07	-.10E+07	0.18E+07	0.12E+07	0.18E+07	-.12E+06	0.18E+07

PEDESTAL DATA

STATION	MASS-X	STIFF-X	DAMP-X	MASS-Y	STIFF-Y	DAMP-Y
3	0.100E-04	0.100E+14	0.100E+00	0.100E-04	0.100E+14	0.100E+00
16	0.100E-04	0.100E+14	0.100E+00	0.100E-04	0.100E+14	0.100E+00
22	0.100E-04	0.100E+14	0.100E+00	0.100E-04	0.100E+14	0.100E+00
27	0.100E-04	0.100E+14	0.100E+00	0.100E-04	0.100E+14	0.100E+00

FOUNDATION SUPPORT DATA

STATION	STIFF-X	DAMP-X	STIFF-Y	DAMP-Y
5	0.100E+33	0.100E+01	0.100E+33	0.100E+01
9	0.100E+33	0.100E+01	0.100E+33	0.100E+01
13	0.100E+33	0.100E+01	0.100E+33	0.100E+01
17	0.100E+33	0.100E+01	0.100E+33	0.100E+01

STATIONS WITH INTERCONNECTION BETWEEN ROTOR AND FOUNDATION, FOR ROTOR AS FOLLOW
3 16 22 27
STATIONS WITH INTERCONNECTION BETWEEN ROTOR AND FOUNDATION, FOR FOUNDATION AS FOLLOW
4 14 17 21

TEST: INSTABILITY, 27 STATIONS, 4 BRG. (3 LEVEL)

FIRST SPEED	LAST SPEED	SPEED INCR.
0.900000E+04	0.100000E+05	0.200000E+03

FREQUENCY RATIOS
0.500000E+00 0.495000E+00 0.490000E+00 0.480000E+00 0.400000E+00
0.350000E+00 0.300000E+00

TABLE 4 RESULTS OF EXAMPLE 2

Case	System	Threshold Speed, rpm	Frequency Ratio, fr	Whirl Speed, rpm	Percentage Difference
(a)	one-level	9910	0.4990	4945	0%
(b)	two-level	9910	0.4990	4945	0%
(c)	three-level, stiff foun.	9873	0.4996	4943	-0.24%
(d)	three-level, soft foun.	9625	0.4998	4811	-2.71%
(e)	two-level, soft pedestal	9732	0.4995	4861	-1.7%

EXAMPLE 3. 300 MW TURBINE-GENERATOR UNIT

The turbine-generator rotor shown in figure 10 operates at 3600 rpm and is mounted in six fluid-film bearings, ranging in size from 200 mm (7.87 in) to 400 mm (15.75 in) shaft diameter. Bearing oil inlet viscosity is 16 cp at 160oF. The bearings are supported on flexible pedestals, mounted upon a stiff foundation, which was effectively a two-level system. The rotor model has 15 stations. The threshold speed of the system was calculated (a) using two plain cylindrical bearings for the generator (i.e., #4 and #5 bearings), and four MFG-Type bearings for the other supports, and (b) using six MFG-Type bearings for all supports.

The results of the stability threshold speed calculations are shown in table 5. For case (a), the threshold speed is 2200 rpm. The corresponding frequency ratio is 0.3518, which corresponds to a whirl speed of 2200 x 0.3677 = 809 rpm. This is close to the first flexible critical speed of the generator, which occurs at 837 rpm. For case (b), no threshold speed was found below 4000 rpm. These results agree with field observations, and demonstrate the capability of MFG-Type bearings for preventing rotor whirl instability.

TABLE 5 RESULTS FOR EXAMPLE 3

Case	System	Threshold Speed, rpm	Frequency Ratio	Whirl Speed Nw=Nth x fr rpm
(a)	Two Plain Cylindrical Brgs. for Generator, Four MFG Type Brgs. for Others	2200	0.3677	809
(b)	Six MFG Type Brgs.	No	No	No

CONCLUSIONS

1. A general-purpose rotordynamics analysis procedure and computer code has been described for the calculation of instability threshold conditions for damped flexible rotor-bearing systems mounted in pedestals, and supported on a distributed mass-elastic flexible foundation.

2. The procedure described has been verified using several established results from the open literature, and close correlation between pedestal threshold speed and whirl frequencies has been found.

3. Application of this procedure to a six-bearing, five-rotor, turbine-generator system has been described. Results were consistent with field observations for this case.

4. The procedure is suitable for evaluating the influence of pedestals and of distributed support foundations on the stability of multi-span rotor systems. The transfer matrix procedure is efficient, and double-precision accuracy is ensured using the complex eigenvalue algorithm described.

ACKNOWLEDGEMENT

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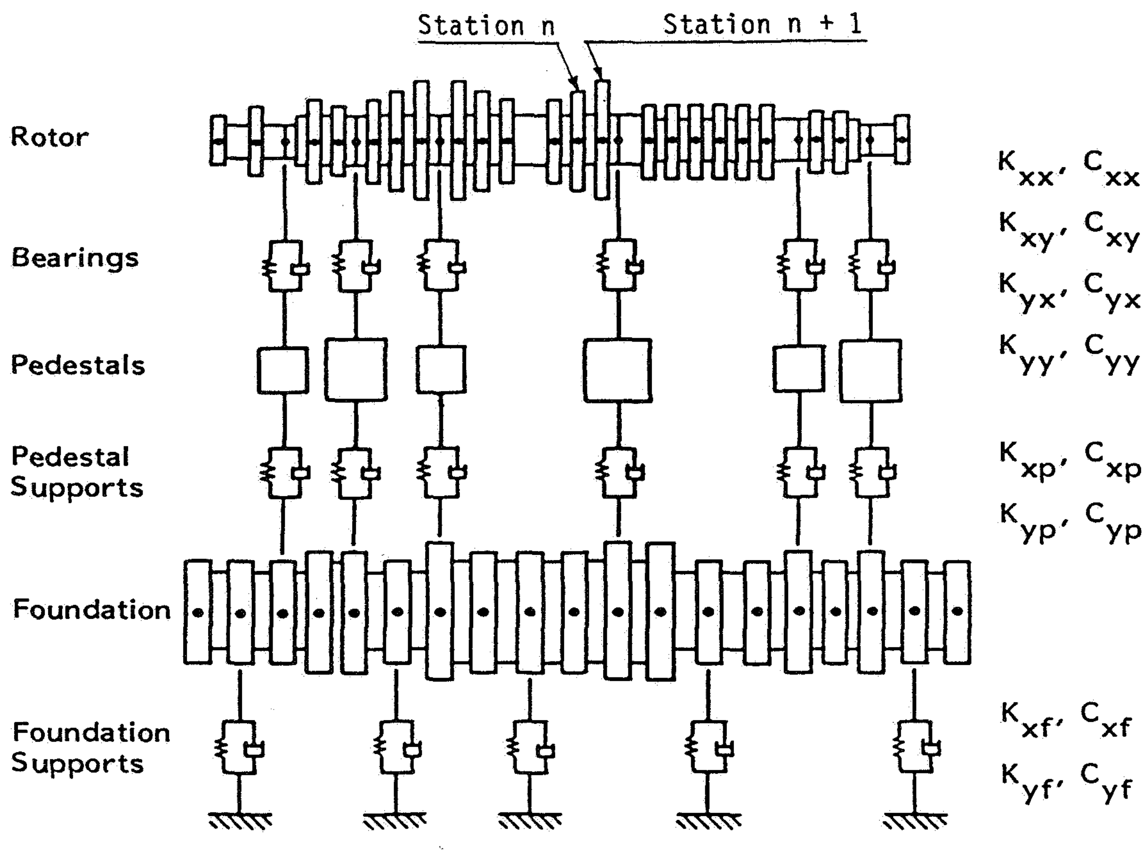
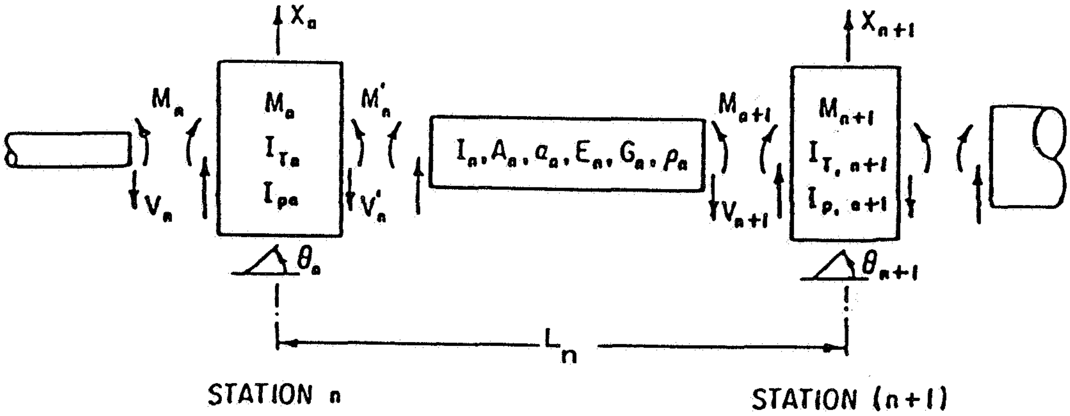
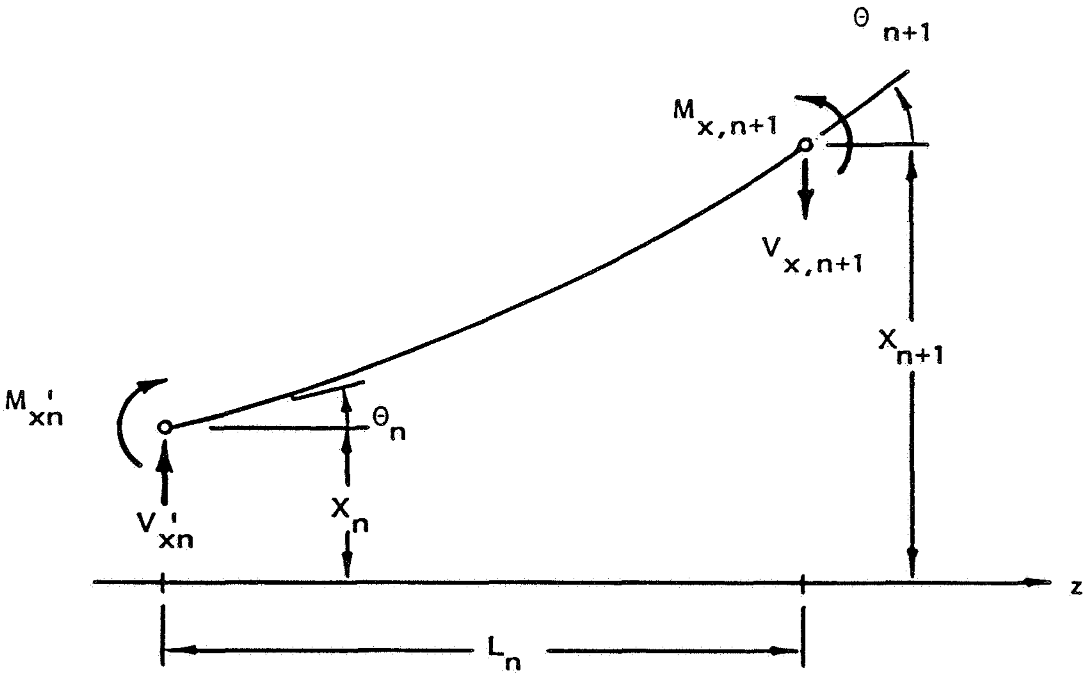


Figure 1 Rotor-Bearing-Pedestal Foundation System



a. End Station and Shaft Nomenclature



b. Shaft Section Between Rotor Stations n and n+1

Figure 2 Convention and Nomenclature for End Stations and Shaft Sections

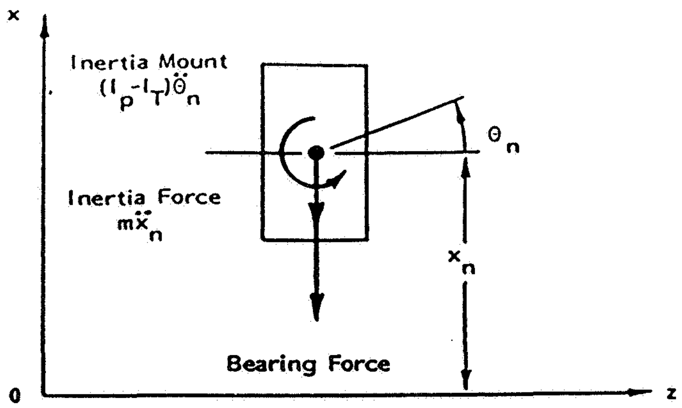
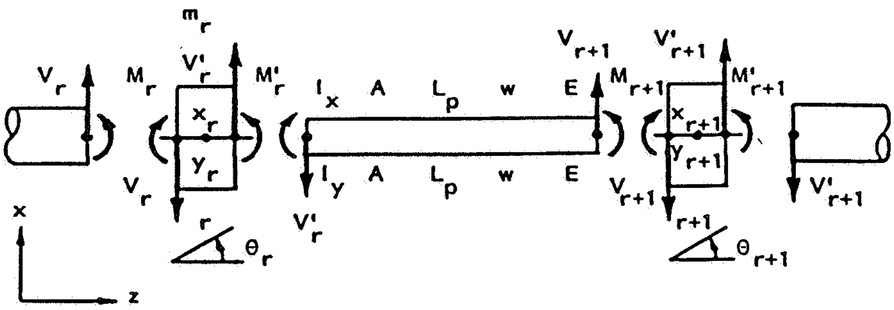
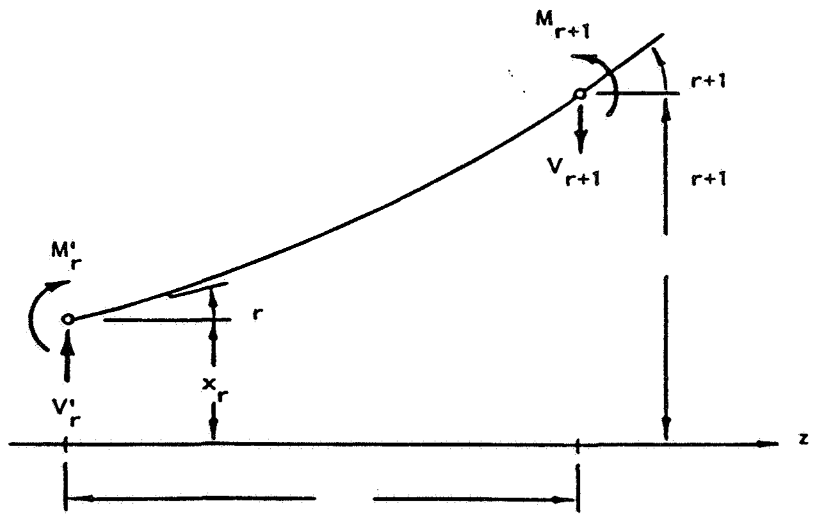


Figure 3   Forces on n-th Rotor Station



a. Notation and Convention for Foundation



b. Moments and Shears on Foundation Section

Figure 4   Notation and Convention for Foundation

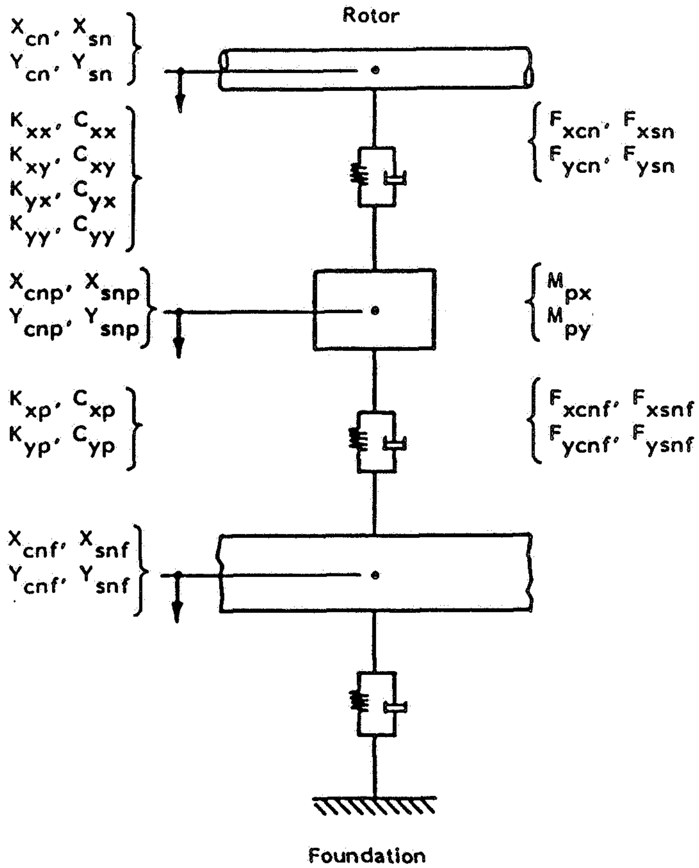


Figure 5 Three-Level Support System for Solution Algorithm

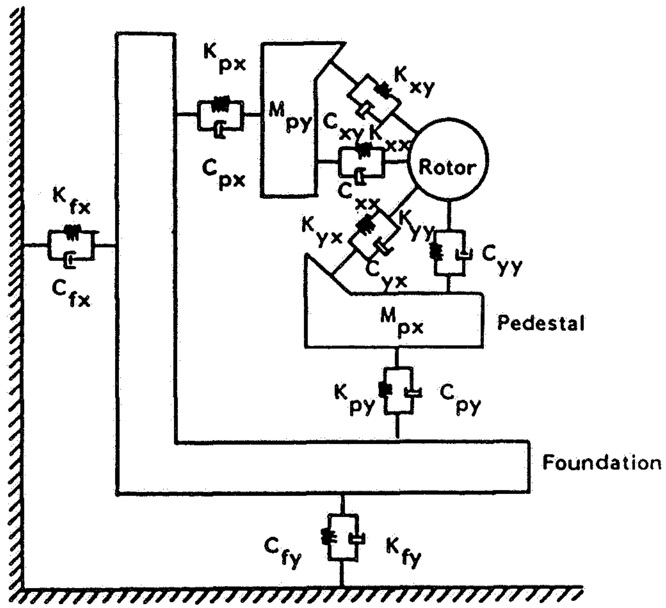


Figure 6 Bearing, Pedestal and Foundation Model for System

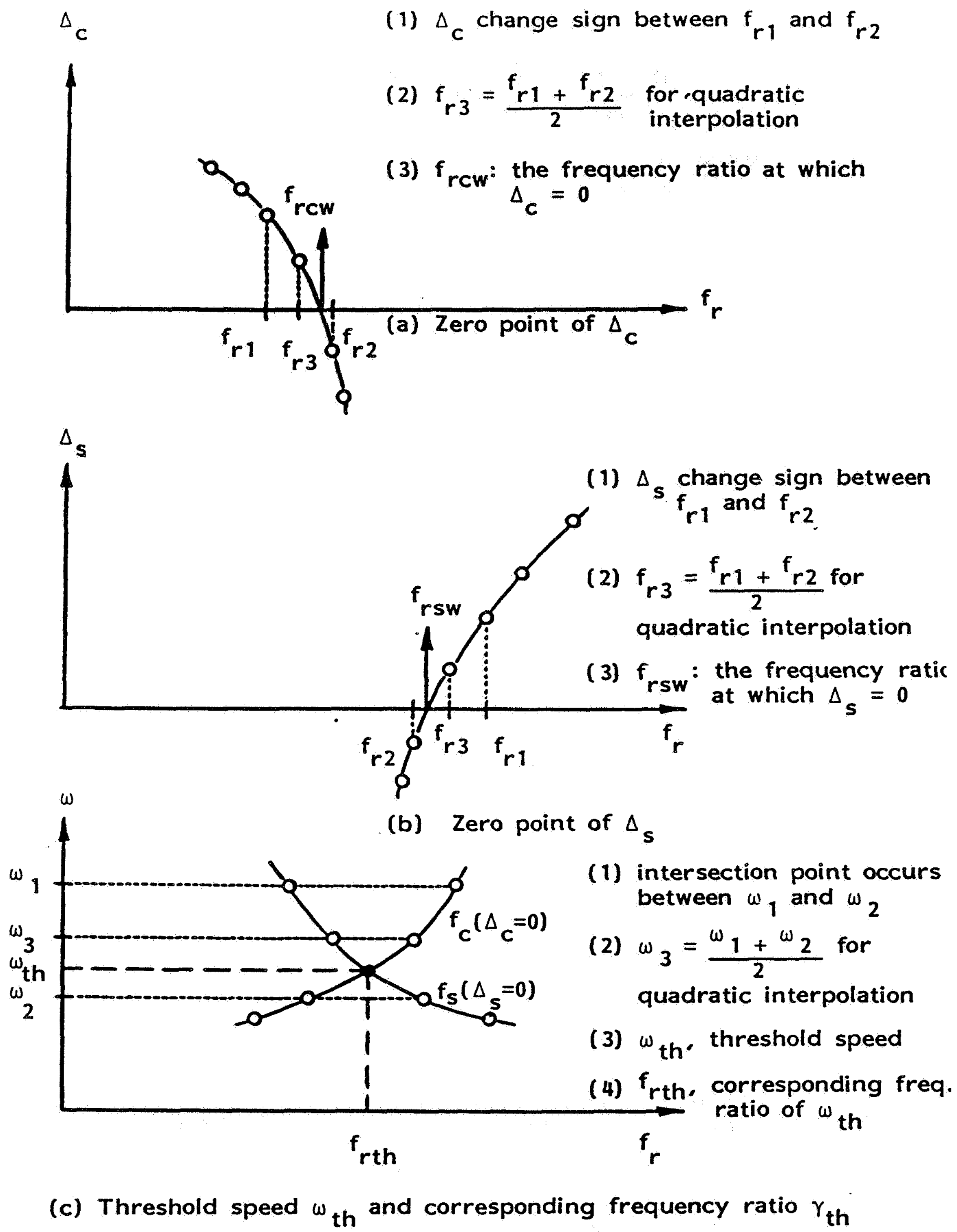


Figure 7 Threshold Speed Determination from the Computer

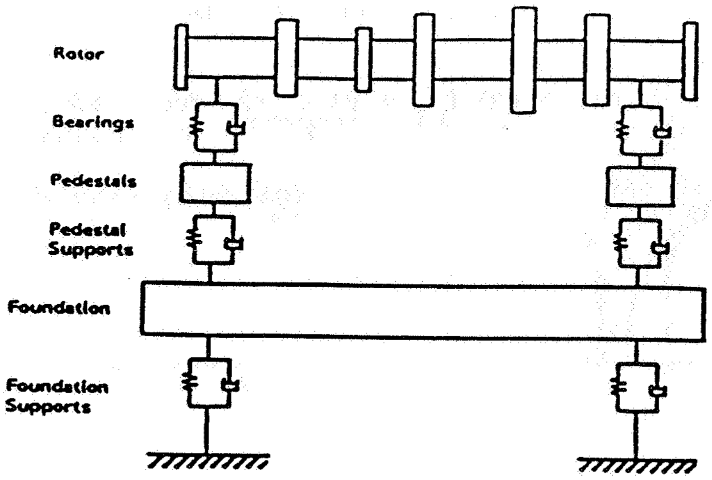


Figure 8 Rigid Rotor System. Example 1. After Lund [4]

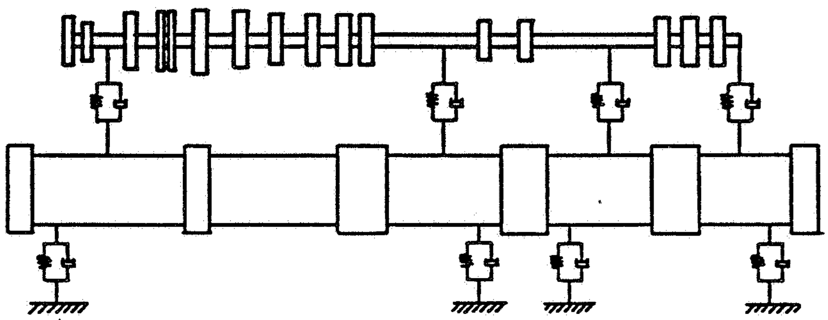


Figure 9 Flexible Rotor in Four Bearings. After Lund [4]

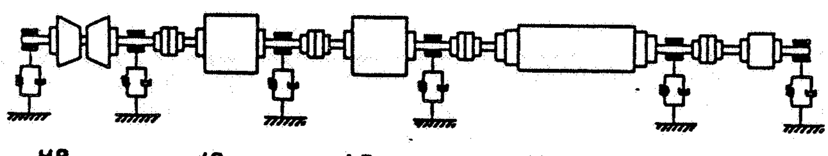


Figure 10 300 Mw Turbine-Generator Rotor System. Five Rotors; Six Bearings